

Effects of Matrix Orientifolding to Two-Loop Effective Action of Bosonic IIB Matrix Model

R. Yoshioka^{a*}

^a *Osaka City University Advanced Mathematical Institute (OCAMI)*

3-3-138, Sugimoto, Sumiyoshi, Osaka, 558-8585, Japan

Abstract

We study the spacetime structures which are described by the IIB matrix model with orientifolding. Matrix orientifolding that preserves supersymmetries yields the mirror image point with respect to a four-dimensional plane for each spacetime point that corresponds to the eigenvalue of the bosonic matrix. In order to consider the upper bound on the distance between two eigenvalues in this model, we calculate the effective action for the eigenvalues up to two-loop. The eigenvalues distribute in a tubular region around the four-dimensional plane.

*e-mail: yoshioka@sci.osaka-cu.ac.jp

1 Introduction

The reduced matrix models are proposed to enable nonperturbative studies of strings [1]-[8]. They are obtained from Yang-Mills theory by the dimensional reduction[9]. The eigenvalues of the matrices represent spacetime points, while the remaining degrees of freedom mediate interactions between the spacetime points. The effective dynamics of the spacetime points is obtained by carrying out the integrations of the off-diagonal elements. The formation of our spacetime have been variously attempted. For example, branched polymers [10], generalised monopoles [11, 12], orbifolding [13, 14] and more have been used. The spontaneous breakdown of Lorentz symmetry to lower dimension due to fermion determinant is also developed [15].

The USp matrix model is introduced as the orientifolding of the IIB matrix model, which preserves the maximal supersymmetry [4, 5]. In [16], we have seen that there is a long distance attraction between the spacetime points up to the one-loop corrections in this model. Moreover it was found that two-body force in the short distance is repulsive by calculations at the model with lower rank matrices, $usp(2)$ and $usp(4)$.

We continued to study the USp matrix model,

$$S = -\frac{1}{4g^2}\text{tr}[v_M, v_N]^2 - \frac{1}{2g^2}\text{tr}\Psi\Gamma^M[v_M, \Psi], \quad (1.1)$$

where $M = 0, 1, \dots, 9$ and Ψ is a ten-dimensional Majorana-Weyl spinor. The matrices v_M take the following form:

$$v_\mu = \begin{pmatrix} M_\mu & N_\mu \\ N_\mu^* & -M_\mu^t \end{pmatrix}, \quad v_m = \begin{pmatrix} A_m & B_m \\ -B_m^* & A_m^t \end{pmatrix}, \quad (1.2)$$

where $\mu = 0, 1, 2, 3, 4, 7$ and $m = 5, 6, 8, 9$ and M_μ and A_m are $N \times N$ Hermitian matrices and $N_\mu(B_m)$ are $N \times N$ (anti-)symmetric. Selecting one of these two representations for each of the matrix coordinates is referred to as matrix orientifolding in this paper. In what follows suppose that the matrices labeled by Greek letters μ, ν, \dots belong to defining representation (we also call this as adjoint representation) of usp Lie algebra and those labeled by Roman letters m, n, \dots belong to antisymmetric representation. This splitting of the representation has taken place in order to preserve $8 + 8$ supersymmetries after the orientifolding of the IIB matrix model with $16 + 16$ supersymmetries and is almost a unique way to choose. By construction, the ten-dimensional Lorentz covariance is broken to four- and six-dimensional ones explicitly. The upshot is that matrix orientifolding inevitably introduces spacetime directional asymmetry. In this paper we restrict, however, our attention to the bosonic part.

We consider the effective dynamics for the eigenvalues of the bosonic matrices, which are obtained by the integrations of the offdiagonal elements. In [17], the author found the upper bound on the extent of spacetime in the bosonic IIB matrix model by calculating the two-loop corrections to the effective action for the eigenvalues. By using the similar prescription, we discuss what feature the spacetime described by the USp matrix model has.

The content of this paper is as follows: In the next section we calculate concretely the two-loop effective action for the eigenvalues. In section three the spacetime constituted by the USp matrix model is discussed.

2 Two-loop corrections of the bosonic USp matrix model

In this section, we consider the bosonic part of the USp matrix model. Then we can arbitrarily choose the number of the bosonic coordinates belonging to either adjoint representation or antisymmetric representation, because this restriction is due to supersymmetry. We denote the number of the directions of the adjoint and antisymmetric representation by D_{ad} and D_{as} , respectively. The action is

$$S_b = -\frac{1}{4g^2} \text{tr}[v_M, v_N]^2, \quad (2.1)$$

We decompose the matrices v_M into the diagonal and the off-diagonal parts,

$$v_M = x_M + \tilde{v}_M. \quad (2.2)$$

The diagonal parts x_M are respectively given by

$$x_\mu = \begin{pmatrix} X_\mu & 0 \\ 0 & -X_\mu \end{pmatrix}, \quad x_m = \begin{pmatrix} X_m & 0 \\ 0 & X_m \end{pmatrix}, \quad (2.3)$$

where

$$X_M = \begin{pmatrix} x_M^1 & & & \\ & x_M^2 & & \\ & & \ddots & \\ & & & x_M^N \end{pmatrix}. \quad (2.4)$$

Since the eigenvalues of the matrix correspond to the spacetime points, the diagonal matrices (2.3) represent the spacetime configuration as shown in Figure 1. Note that the lower half of the diagonal elements is not independent variables and correspond to the

mirror image points with respect to the D_{as} -dimensional plane spanned by the directions of antisymmetric representation. The upper half of the matrices thus represents spacetime.

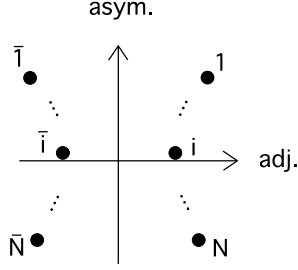


Figure 1: Each node represents the diagonal elements of the matrices x_M , which are regarded as spacetime points and their images in ten dimensional spacetime. Because ten $2N \times 2N$ bosonic matrices appear in the USp matrix model, there are $2N$ points in this figure. Clearly, the spacetime points and their images are symmetric with respect to the four-dimensional plane spanned by the antisymmetric directions.

Using the eigenvalues λ_M of v_M and the off-diagonal elements \tilde{v}_M , we can rewrite the diagonal elements x_M as

$$x_M^{i'} \sim \lambda_M^{i'} - \sum_{j' \neq i'} \frac{\tilde{v}_M^{i'j'} \tilde{v}_M^{j'i'}}{\lambda_M^{i'} - \lambda_M^{j'}} \equiv \lambda_M^{i'} + x'_M, \quad (2.5)$$

up to $O(\tilde{v}^2)$, where $i' = 1, 2, \dots, 2N$. The difference x'_M between the diagonal element and eigenvalue becomes important for higher loop corrections though it can be neglected for one-loop.

The quadratic action for the off-diagonal elements is

$$\begin{aligned} S' &= S_b^{(2)} + S_{\text{gf}}^{(2)} + S_{\text{ghost}}^{(2)} \\ &= \frac{1}{g^2} \sum_{i,j} \left[\{(\lambda_\mu^{ij})^2 + (\lambda_m^{ij})^2\} M_\nu^{ij} M^{\nu ij*} + \{(\tilde{\lambda}_\mu^{ij})^2 + (\lambda_m^{ij})^2\} N_\nu^{ij} N^{\nu ij*} \right. \\ &\quad \left. + \{(\lambda_\mu^{ij})^2 + (\lambda_m^{ij})^2\} A_n^{ij} A^{nij*} + \{(\tilde{\lambda}_\mu^{ij})^2 + (\lambda_m^{ij})^2\} B_n^{ij} B^{nij*} \right] \\ &\quad - \frac{1}{g^2} \text{tr}[\lambda_M, b][\lambda_M, c], \end{aligned} \quad (2.6)$$

where

$$\lambda_M^{ij} = \lambda_M^i - \lambda_M^j, \quad \tilde{\lambda}_M^{ij} = \lambda_M^i + \lambda_M^j. \quad (2.7)$$

Here we have added the following gauge fixing term and the ghost term to the action:

$$S_{\text{gf}} = -\frac{1}{2g^2} \text{tr}[\lambda_M, \tilde{v}_M], \quad S_{\text{ghost}} = -\frac{1}{g^2} \text{tr}[\lambda_M, b][v_M, c]. \quad (2.8)$$

In Eq.(2.6), the quadratic parts $S_{\text{gf}}^{(2)}$ and $S_{\text{ghost}}^{(2)}$ have been included. The ghost c and anti-ghost b belong to adjoint representation,

$$c = \begin{pmatrix} c_{(1)} & c_{(2)} \\ c_{(2)}^* & -c_{(1)}^t \end{pmatrix}, \quad b = \begin{pmatrix} b_{(1)} & b_{(2)} \\ b_{(2)}^* & -b_{(1)}^t \end{pmatrix}, \quad (2.9)$$

where $b_{(1)}$ and $c_{(1)}$ are Hermitian and $b_{(2)}$ and $c_{(2)}$ are symmetric. The propagators can be read off from the quadratic action (2.6) as follows:

$$\begin{aligned} \langle A_M^{*ij} A_N^{kl} \rangle &= g^2 \frac{1}{(\lambda^{ij})^2} \delta^{ik} \delta^{jl} \delta_{MN}, \\ \langle N_\mu^{*ij} N_\nu^{kl} \rangle &= g^2 \frac{1}{(\tilde{\lambda}^{ij})^2} \frac{1}{2} (\delta^{ik} \delta^{jl} + \delta^{il} \delta^{jk}) \delta_{\mu\nu}, \\ \langle B_m^{*ij} B_n^{kl} \rangle &= g^2 \frac{1}{(\tilde{\lambda}^{ij})^2} \frac{1}{2} (\delta^{ik} \delta^{jl} - \delta^{il} \delta^{jk}) \delta_{mn}, \\ \langle c_{(1)}^{ij} b_{(1)}^{kl} \rangle &= g^2 \frac{1}{(\lambda^{ij})^2} \delta^{il} \delta^{jk}, \\ \langle c_{(2)}^{ij} b_{(2)}^{kl} \rangle &= g^2 \frac{1}{(\tilde{\lambda}^{ij})^2} \frac{1}{2} (\delta^{ik} \delta^{jl} + \delta^{il} \delta^{jk}), \end{aligned} \quad (2.10)$$

where $\langle \rangle$ represents expectation value defined by

$$\langle \mathcal{O} \rangle = \frac{\int \mathcal{O} e^{-S'}}{\int e^{-S'}}, \quad (2.11)$$

and $A_M = (M_\mu, A_m)$ whose eigenvalues are ten-dimensional spacetime coordinates and $(\lambda^{ij})^2 = (\lambda_M^i - \lambda_M^j)^2$ and $(\tilde{\lambda}^{ij})^2 = (\lambda_\mu^i - \lambda_\mu^j)^2 + (\lambda_m^i + \lambda_m^j)^2$, which correspond to each distance between the eigenvalues. The former $(\lambda^{ij})^2$ is the second power of distance between two spacetime points and the latter $(\tilde{\lambda}^{ij})^2$ is that between a spacetime point and a mirror image point. The interaction part is

$$\begin{aligned} S_{\text{int}} &= -\frac{1}{g^2} \text{tr}[\lambda_M, \tilde{v}_M][\tilde{v}_M, \tilde{v}_N] - \frac{1}{4g^2} \text{tr}[\tilde{v}_M, \tilde{v}_N]^2 - \frac{1}{g^2} \text{tr}[x'_M, \tilde{v}_N][\lambda_M, \tilde{v}_N] \\ &\quad + \frac{1}{g^2} \text{tr}[x'_M, \tilde{v}_M][\lambda_N, \tilde{v}_N] - \frac{1}{g^2} \text{tr}[\lambda_M, b][\tilde{v}_M, c] - \frac{1}{g^2} \text{tr}[\lambda_M, b][x'_M, c]. \end{aligned} \quad (2.12)$$

The action (2.1) is also written as

$$S_b = -\frac{1}{2g^2} \text{tr}[A_M, A_N]^2 + \dots. \quad (2.13)$$

The first term is the bosonic action of the $D = D_{ad} + D_{as} = 10$ IIB matrix model and the remaining terms appear by the effect of orientifolding, which is our interest. The two-loop effective action, therefore, is written as

$$W_2(\lambda) = g^2 W_2^{\text{IIB}}(\lambda) + g^2 W_2'(\lambda). \quad (2.14)$$

Here the first term is the two-loop effective action for the IIB matrix model [17],

$$W_2^{\text{IIB}}(\lambda) = \frac{1}{2}(D-2)^2 I_1 - \frac{1}{2}D(3D-7)I_2 - 2(D-2)I_3, \quad (2.15)$$

where

$$\begin{aligned} I_1 &\equiv \sum_{i,j,k,j \neq k} \frac{1}{(\lambda^{ij})^2 (\lambda^{ik})^2}, \\ I_2 &\equiv \sum_{i,j} \frac{1}{(\lambda^{ij})^4}, \\ I_3 &\equiv \sum_{i,j,k,j \neq k} \frac{1}{(\lambda^{ij})^2 (\lambda^{ik})^2} \frac{\lambda_M^{ij}}{\lambda_M^{ik}}. \end{aligned} \quad (2.16)$$

In what follows we calculate concretely the remaining part of the two-loop effective action. In this section, we pay attention to the action constituted by only the matrices of the adjoint representation, M_μ , N_μ and the ghosts. The part including the matrices of the antisymmetric representation is calculated in Appendix A. The action that we need now is

$$\begin{aligned} S_{int}^{ad} = & \sum_{i,j} \sum_{\mu,\nu} \left\{ -\frac{1}{g^2} \left[2\lambda_\mu^{ij} M_\nu^{ij} E_{\mu\nu}^{ji} - \tilde{\lambda}_\mu^{ij} (N_\nu^{ij} F_{\mu\nu}^{ij*} + N_\nu^{ij*} F_{\mu\nu}^{ij}) \right] - \frac{1}{2g^2} [E_{\mu\nu}^{ij} E_{\mu\nu}^{ji} - F_{\mu\nu}^{ij} F_{\mu\nu}^{ij*}] \right. \\ & - \frac{1}{g^2} \left[2\lambda_\mu^{ij} b_{(1)\mu}^{ij} E_{(c)\mu}^{ji} - \tilde{\lambda}_\mu^{ij} (b_{(2)\mu}^{ij} F_{(c)\mu}^{ij*} + b_{(2)\mu}^{ij*} F_{(c)\mu}^{ij}) \right] \Big\} \\ & - \frac{4}{g^2} \left\{ \sum_{i,j,k \neq i} \left[\frac{\lambda_\mu^{ij}}{\lambda_\mu^{ik}} M_\mu^{ik} M_\mu^{ki} M_\nu^{ij} M_\nu^{ji} + \frac{\tilde{\lambda}_\mu^{ij}}{\lambda_\mu^{ik}} M_\mu^{ik} M_\mu^{ki} N_\nu^{ij} N_\nu^{ji*} \right] \right. \\ & \left. + \sum_{ijk} \left[\frac{\tilde{\lambda}_\mu^{ij}}{\lambda_\mu^{ik}} N_\mu^{ik} N_\mu^{ki*} N_\nu^{ij} N_\nu^{ji*} + \frac{\lambda_\mu^{ij}}{\tilde{\lambda}_\mu^{ik}} N_\mu^{ik} N_\mu^{ki*} M_\nu^{ij} M_\nu^{ji} \right] \right\} \\ & + \frac{2}{g^2} \left\{ \sum_{i,j,k \neq i} \left[\frac{\lambda_\nu^{ij}}{\lambda_\nu^{ik}} M_\mu^{ik} M_\mu^{ki} (M_\mu^{ij} M_\nu^{ji} + M_\mu^{ji} M_\nu^{ij}) + \frac{\tilde{\lambda}_\nu^{ij}}{\lambda_\nu^{ik}} M_\mu^{ik} M_\mu^{ki} (N_\mu^{ij} N_\nu^{ij*} + N_\mu^{ij*} N_\nu^{ij}) \right] \right. \\ & \left. + \sum_{ijk} \left[\frac{\tilde{\lambda}_\nu^{ij}}{\lambda_\nu^{ik}} N_\mu^{ik} N_\mu^{ki*} (N_\mu^{ij} N_\nu^{ij*} + N_\mu^{ij*} N_\nu^{ij}) + \frac{\lambda_\nu^{ij}}{\tilde{\lambda}_\nu^{ik}} M_\mu^{ik} M_\mu^{ki} (N_\mu^{ij} N_\nu^{ij*} + N_\mu^{ij*} N_\nu^{ij}) \right] \right\} \\ & - \frac{2}{g^2} \left\{ \sum_{i,j,k \neq i} \left[\frac{\lambda_\mu^{ij}}{\lambda_\mu^{ik}} M_\mu^{ik} M_\mu^{ki} (b_{(1)}^{ij} c_{(1)}^{ji} + b_{(1)}^{ji} c_{(1)}^{ij}) + \frac{\tilde{\lambda}_\mu^{ij}}{\lambda_\mu^{ik}} M_\mu^{ik} M_\mu^{ki} (b_{(2)}^{ij} c_{(2)}^{ij*} + b_{(2)}^{ij*} c_{(2)}^{ij}) \right] \right. \\ & \left. + \sum_{ijk} \left[\frac{\tilde{\lambda}_\mu^{ij}}{\lambda_\mu^{ik}} N_\mu^{ik} N_\mu^{ki*} (b_{(2)}^{ij} c_{(2)}^{ij*} + b_{(2)}^{ij*} c_{(2)}^{ij}) + \frac{\lambda_\mu^{ij}}{\tilde{\lambda}_\mu^{ik}} N_\mu^{ik} N_\mu^{ki*} (b_{(1)}^{ij} c_{(1)}^{ji} + b_{(1)}^{ji} c_{(1)}^{ij}) \right] \right\}, \end{aligned} \quad (2.17)$$

where

$$E_{\mu\nu}^{ij} = [M_\mu, M_\nu]^{ij} + (N_\mu N_\nu^*)^{ij} - (N_\nu N_\mu^*)^{ij}, \quad (2.18)$$

$$F_{\mu\nu}^{ij} = [(M_\mu N_\nu)^{ij} + (M_\mu n_\nu)^{ji}] - [(M_\nu N_\mu)^{ij} + (M_\nu n_\mu)^{ji}], \quad (2.19)$$

$$E_{(c)\mu}^{ij} = [M_\mu, c_{(1)}]^{ij} + (N_\mu c_{(2)}^*)^{ij} - (c_{(2)} N_\mu^*)^{ij}, \quad (2.20)$$

$$F_{(c)\mu}^{ij} = [(M_\mu c_{(2)})^{ij} + (M_\mu c_{(2)})^{ji}] - [(c_{(1)} N_\mu)^{ij} + (c_{(1)} N_\mu)^{ji}]. \quad (2.21)$$

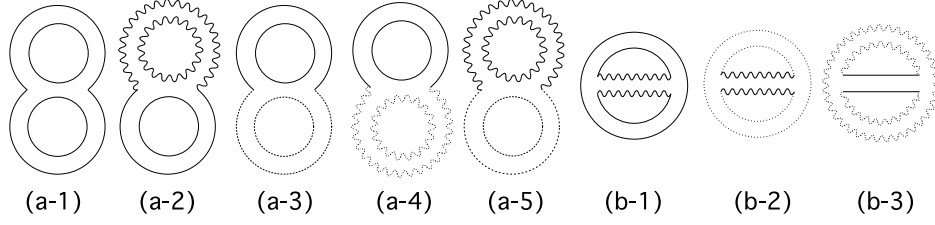


Figure 2: two-loop planar diagrams.

The two-loop planar diagrams are shown in Figure 2. The solid line and the wavy line represent the propagator of N_μ^{ij} and M_μ^{ij} , respectively. Similarly, the dashed solid line and wavy line correspond to the ghosts $b_{(2)}, c_{(2)}$ and $b_{(1)}, c_{(1)}$. The diagram (a-1) in Figure 2 is evaluated as

$$\begin{aligned} (a-1) &= \left\langle \sum_{i,j,k,l} \sum_{\mu,\nu} (N_\mu^{ik} N_\nu^{kj*} N_\mu^{jl} N_\nu^{li*} - N_\mu^{ik*} N_\mu^{kj} N_\nu^{jl*} N_\nu^{li}) \right. \\ &\quad \left. + \sum_{i,j,k} \sum_{\mu,\nu} 4 \frac{\tilde{\lambda}_\mu^{ij}}{\tilde{\lambda}_{ik}^\mu} N_\mu^{ik} N_\mu^{ki*} N_\nu^{ij} N_\nu^{ji*} - \sum_{i,j,k} \sum_{\mu,\nu} 2 \frac{\tilde{\lambda}_\nu^{ij}}{\tilde{\lambda}_{ik}^\mu} N_\mu^{ik} N_\nu^{ik*} (N_\mu^{ij} N_\nu^{ij*} + N_\mu^{ij*} N_\nu^{ij}) \right\rangle \\ &= -\frac{1}{2} D_{ad} (D_{ad} - 1) J_1^+ + \frac{3}{2} D_{ad} (D_{ad} - 1) J_2^+ + 2(D_{ad} - 1) J_3, \end{aligned} \quad (2.22)$$

where

$$\begin{aligned} J_1^\pm &= \sum_{i,j,k \neq j} \frac{1}{2} \frac{1}{(\tilde{\lambda}^{ij})^2 (\tilde{\lambda}^{ik})^2} (1 \pm \delta^{ij} \pm \delta^{ik}), \\ J_2^\pm &= \sum_{i,j} \frac{1}{2} \frac{1}{(\tilde{\lambda}^{ij})^4} (1 + \delta^{ij} \pm 2\delta^{ij}), \\ J_3 &= \sum_{i,j,k \neq j} \sum_\mu \frac{1}{2} \frac{\tilde{\lambda}_\mu^{ij}}{\tilde{\lambda}_{ik}^\mu} \frac{1}{(\tilde{\lambda}^{ij})^2 (\tilde{\lambda}^{ik})^2} (1 + \delta^{ik} + \delta^{ij}). \end{aligned} \quad (2.23)$$

Similarly, the diagrams (a-2) \sim (b-3) are evaluated as

$$(a-2) = -2D_{ad}(D_{ad} - 1)K_1^{(1)} + 4(D_{ad} - 1)K_3^{(1)} + 4(D_{ad} - 1)\tilde{K}_3^{(1)}, \quad (2.24)$$

$$(a-3) = -2(D_{ad}J_2^+ + J_3), \quad (2.25)$$

$$(a-4) = -4K_3^{(1)}, \quad (2.26)$$

$$(a-5) = -4\tilde{K}_3^{(1)}, \quad (2.27)$$

$$(b-1) = 2(D_{ad} - 1)L_1^{(1)} + 2(D_{ad} - 1)L_2^{(1)} + 4(D_{ad} - 1)\tilde{L}_2^{(1)}, \quad (2.28)$$

$$(b-2) = -2L_2^{(1)}, \quad (2.29)$$

$$(b-3) = L_3, \quad (2.30)$$

where

$$\begin{aligned} K_1^\pm &= \sum_{i,j,k \neq j} \frac{1}{2} \frac{1}{(\lambda^{jk})^2 (\tilde{\lambda}^{ij})^2} (1 + \delta^{ik} \pm \delta^{ij}), \\ K_3^{(1)} &= \sum_{i,j,k \neq i} \sum_{\mu} \frac{1}{2} \frac{\lambda_{\mu}^{ik}}{\tilde{\lambda}_{\mu}^{ij}} \frac{1}{(\lambda^{ik})^2 (\tilde{\lambda}^{ij})^2} (1 + \delta^{ij}), \\ \tilde{K}_3^{(1)} &= \sum_{i,j,k \neq i} \sum_{\mu} \frac{1}{2} \frac{\tilde{\lambda}_{\mu}^{ij}}{\lambda_{\mu}^{ik}} \frac{1}{(\lambda^{ik})^2 (\tilde{\lambda}^{ij})^2} (1 + \delta^{ij}), \\ L_1^{(1)} &= \sum_{i,j,k \neq i} \sum_{\mu} \frac{1}{2} \frac{(\lambda_{\mu}^{ik})^2}{(\lambda^{ik})^2 (\tilde{\lambda}^{ij})^2 (\tilde{\lambda}^{kj})^2} (1 + \delta^{ij} + \delta^{jk}), \\ L_2^{(1)} &= \sum_{i,j,k \neq i} \sum_{\mu} \frac{1}{2} \frac{\tilde{\lambda}_{\mu}^{ij} \tilde{\lambda}_{\mu}^{kj}}{(\lambda^{ik})^2 (\tilde{\lambda}^{ij})^2 (\tilde{\lambda}^{jk})^2} (1 + \delta^{ij} + \delta^{jk}), \\ \tilde{L}_2^{(1)} &= \sum_{i,j,k \neq i} \sum_{\mu} \frac{1}{2} \frac{\tilde{\lambda}_{\mu}^{ij} \tilde{\lambda}_{\mu}^{kj}}{(\lambda^{ik})^2 (\tilde{\lambda}^{ij})^2 (\tilde{\lambda}^{jk})^2} (1 + \delta^{ij} + \delta^{jk}), \\ L_3 &= \sum_{i,j} \sum_{\mu} \frac{1}{4} \frac{\lambda_{\mu}^{ij} \tilde{\lambda}_{\mu}^{ij}}{(\lambda^{ij})^2 (\tilde{\lambda}^{ij})^2 (\tilde{\lambda}^{jj})^2} (1 + \delta^{ij} + \delta^{jk}). \end{aligned} \quad (2.31)$$

Since the two-loop effective action is given by summing up all corrections and flipping the sign, we obtain the adjoint part,

$$\begin{aligned} W_2^{ad}(\lambda) &= \frac{1}{2} D_{ad}(D_{ad} - 1)J_1^+ - \frac{1}{2} D_{ad}(3D_{ad} - 7)J_2^+ - 2(D_{ad} - 2)J_3 \\ &\quad + 2D_{ad}(D_{ad} - 1)K_1^+ - 4(D_{ad} - 2)K_3^{(1)} - 4(D_{ad} - 2)\tilde{K}_3^{(1)} \\ &\quad - 2(D_{ad} - 1)L_1^{(1)} - 2(D_{ad} - 2)L_2^{(1)} - 2(D_{ad} - 1)\tilde{L}_2^{(1)} - L_3. \end{aligned} \quad (2.32)$$

When the result calculated in the Appendix A is included, we can get the orientifolding effect part of the two-loop effective action,

$$W'(\lambda) = W_2^{ad}(\lambda) + W_2^{as}(\lambda) + W_2^{int}(\lambda), \quad (2.33)$$

where $W_2^{as}(\lambda)$ and $W_2^{int}(\lambda)$ are given in equation (A.8) and (A.15).

3 Discussion

In the last section, the two-loop effective action have been calculated. First, according to the procedure used in the paper [17], in order to discuss the extent of the spacetime described by the USp matrix model, we estimate the order of magnitude of the two-loop correction. Then, we can determine the upper bound of the distance between two eigenvalues in the USp matrix model. The order of magnitude of the one-loop correction is

$$(\text{one-loop}) \sim O(N^2), \quad (3.1)$$

and that of the two-loop corrections which is obtained in the last section is

$$(\text{two-loop}) \sim O\left(N^2 \frac{g^2 N}{R^4}\right). \quad (3.2)$$

where R is the expectation value of the meanvalue of the distance between the spacetime points. The two-loop corrections can be neglected for $R > \sqrt{g}N^{\frac{1}{4}}$. Thus it is enough that we think of only the attraction by the one-loop corrections in this region. As there are mirror image points with respect to plane spanned by the antisymmetric directions, the points are attracted to this D_{as} -dimensional plane. The spacetime points, therefore, are restricted in the region whose distance from the antisymmetric plane in Figure 3 within $\sqrt{g}N^{\frac{1}{4}}$.

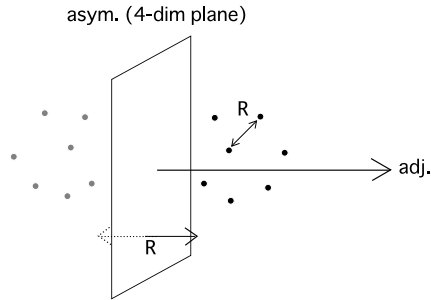


Figure 3: The points are attracted to the plane spanned by the antisymmetric directions due to dominant one-loop effect when the distance of the spacetime point from the plane is large.

We must take supersymmetry into account in this discussion. However although supersymmetry may adjust the two-loop corrections, the above obtained upper bound is

certain. In addition, since supersymmetry demands $D_{ad} = 6$ and $D_{as} = 4$, the antisymmetric plane is four-dimensional.

On the other hand, for each spacetime point, there is always a mirror image point with respect to the plane spanned by the four antisymmetric directions by construction. In [16] we found that two-body interaction between a spacetime point and its mirror image point is described by the six-dimensional $SU(2)$ matrix model. In fact, in the two-loop effective action (2.33), the interaction term between the point i and its mirror image point \bar{i} , which contain the factor $(\tilde{\lambda}^{ii})^4 = (2\lambda_\mu^i)^4$ is written by

$$W'(\lambda)|_{i-\bar{i} \text{ interaction}} = -\frac{1}{2}D_{ad}(3D_{ad} - 7)\frac{1}{(2\lambda^i)^4}, \quad (3.3)$$

which is the same as the interaction term between the eigenvalues λ_μ^i and $-\lambda_\mu^i$ in D_{ad} -dimensional $SU(2)$ matrix model. The number of the coordinates belonging to the antisymmetric representation D_{as} is not related to this interaction. In the $SU(2)$ model, the expectation value of the distance between these two eigenvalues is the quantity of the order of Planck length [18]. This result suggests that the eigenvalues distribute in a tubular region around the four directions of antisymmetric representation.

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A two-loop corrections

In this appendix, we calculate the remaining effective action. The action constituted by the matrices of the antisymmetric representation only is

$$\begin{aligned}
S_{as}^{\text{int}} = & \sum_{i,j} \sum_{m,n} \left\{ -\frac{1}{g^2} [2\lambda_m^{ij} A_n^{ij} E_{mn}^{ji} - \lambda_m^{ij} (B_n^{ij} F_{mn}^{ij*} + B_n^{ij*} F_{mn}^{ij})] - \frac{1}{2g^2} [E_{mn}^{ij} E_{mn}^{ji} - F_{mn}^{ij} F_{mn}^{ij*}] \right. \\
& - \frac{1}{g^2} [2\lambda_m^{ij} b_{(1)}^{ij} E_{(c)m}^{ji} + \lambda_m^{ij} (b_{(2)}^{ij} F_{(c)m}^{ij*} + b_{(2)}^{ij*} F_{(c)m}^{ij})] \Big\} \\
& - \frac{4}{g^2} \left\{ \sum_{ij,k \neq i} \left[\frac{\lambda_m^{ij}}{\lambda_{ik}^m} A_m^{ik} A_m^{ki} A_n^{ij} A_n^{ji} - \frac{\lambda_m^{ij}}{\lambda_{ik}^m} A_m^{ik} A_m^{ki} B_n^{ij} B_n^{ji*} \right] \right. \\
& + \sum_{ij,k \neq i} \left[\frac{\lambda_m^{ij}}{\lambda_{ik}^m} B_m^{ik} B_m^{ki*} B_n^{ij} B_n^{ji*} - \frac{\lambda_m^{ij}}{\lambda_{ik}^m} B_m^{ik} B_m^{ki*} A_n^{ij} A_n^{ji} \right] \Big\} \\
& + \frac{2}{g^2} \left\{ \sum_{ij,k \neq i} \left[\frac{\lambda_n^{ij}}{\lambda_{ik}^n} A_m^{ik} A_m^{ki} (A_m^{ij} A_n^{ji} + A_m^{ji} A_n^{ij}) + \frac{\lambda_n^{ij}}{\lambda_{ik}^n} A_m^{ik} A_m^{ki} (B_m^{ij} B_n^{ij*} + B_m^{ij*} B_n^{ij}) \right] \right. \\
& + \sum_{ijk} \left[\frac{\lambda_n^{ij}}{\lambda_{ik}^n} B_m^{ik} B_m^{ki*} (B_m^{ij} B_n^{ij*} + B_m^{ij*} B_n^{ij}) + \frac{\lambda_n^{ij}}{\lambda_{ik}^n} B_m^{ik} B_m^{ki*} (A_m^{ij} A_n^{ji} + A_m^{ji} A_n^{ij}) \right] \Big\} \\
& - \frac{2}{g^2} \left\{ \sum_{ij,k \neq i} \left[\frac{\lambda_m^{ij}}{\lambda_{ik}^m} A_m^{ik} A_m^{ki} (b_{(1)}^{ij} c_{(1)}^{ji} + b_{(1)}^{ji} c_{(1)}^{ij}) + \frac{\lambda_m^{ij}}{\lambda_{ik}^m} A_m^{ik} A_m^{ki} (b_{(2)}^{ij} c_{(2)}^{ij*} + b_{(2)}^{ij*} c_{(2)}^{ij}) \right] \right. \\
& + \sum_{ijk} \left[\frac{\lambda_m^{ij}}{\lambda_{ik}^m} B_m^{ik} B_m^{ki*} (b_{(2)}^{ij} c_{(2)}^{ij*} + b_{(2)}^{ij*} c_{(2)}^{ij}) + \frac{\lambda_m^{ij}}{\lambda_{ik}^m} B_m^{ik} B_m^{ki*} (b_{(1)}^{ij} c_{(1)}^{ji} + b_{(1)}^{ji} c_{(1)}^{ij}) \right] \Big\}
\end{aligned} \tag{A.1}$$

where

$$E_{mn}^{ij} = [A_m, A_n]^{ij} - (B_m B_n^*)^{ij} + (B_n B_m^*)^{ij} \tag{A.2}$$

$$F_{mn}^{ij} = [(A_m B_n)^{ij} + (A_m B_n)^{ji}] - [(A_n B_m)^{ij} + (A_n B_m)^{ji}] \tag{A.3}$$

$$E_{(c)m}^{ij} = [A_m, c_{(1)}]^{ij} + (B_m c_{(2)}^*)^{ij} - (c_{(2)} B_m^*)^{ij} \tag{A.4}$$

$$F_{(c)m}^{ij} = [(A_m c_{(2)})^{ij} + (A_m c_{(2)})^{ji}] - [(c_{(1)} B_m)^{ij} + (c_{(1)} B_m)^{ji}]. \tag{A.5}$$

The two-loop planar diagrams are the same as Figure 2. However, the solid line and the wavy line represent the propagator of B_m^{ij} and A_m^{ij} , respectively in this case. We can

obtain the following results from each diagram in Figure 2;

$$\begin{aligned}
(\text{a-1}) &= -\frac{1}{2}D_{as}(D_{as}-1)J_1^- + \frac{3}{2}D_{as}(D_{as}-1)J_2^- + 2(D_{as}-1)J_3' \\
(\text{a-2}) &= -2D_{as}(D_{as}-1)K_1^- + 4(D_{as}-1)K_3^{(2)} + 4(D_{as}-1)K_3'^{(2)} \\
(\text{a-3}) &= -2(D_{as}J_2^- + J_3') \\
(\text{a-4}) &= -4K_3^{(2)} \\
(\text{a-5}) &= -4\tilde{K}_3^{(2)} \\
(\text{b-1}) &= 6(D_{as}-1)L_1^{(2)} + 2(D_{as}-1)L_2^{(2)} \\
(\text{b-2}) &= -2L_2^{(2)} \\
(\text{b-3}) &= L_2^{(2)}
\end{aligned} \tag{A.6}$$

where

$$\begin{aligned}
J_3' &= \sum_{i,j,k \neq j} \sum_m \frac{1}{2} \frac{\lambda_m^{ij}}{\lambda_m^{ik}} \frac{1}{(\tilde{\lambda}^{ij})^2 (\tilde{\lambda}^{ik})^2}, \\
K_3^{(2)} &= \sum_{j,k,i \neq j,k} \sum_m \frac{1}{2} \frac{\lambda_m^{ik}}{\lambda_m^{ij}} \frac{1}{(\lambda^{ik})^2 (\tilde{\lambda}^{ij})^2}, \\
\tilde{K}_3^{(2)} &= \sum_{i,j,k \neq i} \sum_m \frac{1}{2} \frac{\lambda_m^{ij}}{\lambda_m^{ik}} \frac{1}{(\lambda^{ik})^2 (\tilde{\lambda}^{ij})^2}, \\
L_1^{(2)} &= \sum_{i,j,k \neq i} \sum_m \frac{1}{2} \frac{(\lambda_m^{ik})^2}{(\lambda^{ik})^2} \frac{1}{(\tilde{\lambda}^{ij})^2 (\tilde{\lambda}^{kj})^2} (1 - \delta^{ij} - \delta^{jk}), \\
L_2^{(2)} &= \sum_{i,j,k \neq i} \sum_m \frac{1}{2} \frac{\lambda_m^{ij} \lambda_m^{kj}}{(\lambda^{ik})^2 (\tilde{\lambda}^{ij})^2 (\tilde{\lambda}^{jk})^2},
\end{aligned} \tag{A.7}$$

Therefore we obtain

$$\begin{aligned}
W_2^{as}(\lambda) &= \frac{1}{2}D_{as}(D_{as}-1)J_1^- - \frac{1}{2}D_{as}(3D_{as}-7)J_2^- - 2(D_{as}-2)J_3' \\
&\quad + 2D_{as}(D_{ad}-1)K_1^- - 4(D_{as}-2)K_3^{(2)} - 4(D_{as}-2)K_3'^{(2)} \\
&\quad - 6(D_{as}-1)L_1^{(2)} - (2D_{as}-3)L_2^{(3)}.
\end{aligned} \tag{A.8}$$

Finally, the remainder of the interaction terms are

$$\begin{aligned}
S'^{\text{int}} = & \sum_{i,j} \sum_{m,n} \left\{ -\frac{1}{g^2} \left[2\lambda_{\mu}^{ij} A_n^{ij} E_{\mu n}^{ji} + \tilde{\lambda}_{\mu}^{ij} (B_n^{ij} F_{\mu n}^{ij*} + B_n^{ij*} F_{\mu n}^{ij}) \right] - \frac{1}{2g^2} [E_{\mu n}^{ij} E_{\mu n}^{ji} + F_{\mu n}^{ij} F_{\mu n}^{ij*}] \right. \\
& - \frac{1}{g^2} [2\lambda_m^{ij} M_{\nu}^{ij} E_{m\nu}^{ji} + \lambda_m^{ij} (N_{\nu}^{ij} F_{m\nu}^{ij*} + N_{\nu}^{ij*} F_{m\nu}^{ij})] - \frac{1}{2g^2} [E_{m\nu}^{ij} E_{m\nu}^{ji} + F_{m\nu}^{ij} F_{m\nu}^{ij*}] \left. \right\} \\
& - \frac{4}{g^2} \left\{ \sum_{i,j,k \neq i} \left[\frac{\lambda_{\mu}^{ij}}{\lambda_{\mu}^{ik}} M_{\mu}^{ik} M_{\mu}^{ki} A_n^{ij} A_n^{ji} - \frac{\tilde{\lambda}_{\mu}^{ij}}{\lambda_{\mu}^{ik}} M_{\mu}^{ik} M_{\mu}^{ki} B_n^{ij} B_n^{ji*} \right] \right. \\
& - \sum_{i,j,k \neq i} \left[\frac{\tilde{\lambda}_{\mu}^{ij}}{\tilde{\lambda}_{\mu}^{ik}} N_{\mu}^{ik} N_{\mu}^{ki*} B_n^{ij} B_n^{ji*} - \frac{\lambda_{\mu}^{ij}}{\tilde{\lambda}_{\mu}^{ik}} N_{\mu}^{ik} N_{\mu}^{ki*} A_n^{ij} A_n^{ji} \right] \left. \right\} \\
& - \frac{4}{g^2} \left\{ \sum_{i,j,k \neq i} \left[\frac{\lambda_m^{ij}}{\lambda_m^{ik}} A_m^{ik} A_m^{ki} M_{\nu}^{ij} M_{\nu}^{ji} + \frac{\lambda_m^{ij}}{\lambda_m^{ik}} A_m^{ik} A_m^{ki} N_{\nu}^{ij} N_{\nu}^{ji*} \right] \right. \\
& - \sum_{i,j,k \neq i} \left[\frac{\lambda_m^{ij}}{\lambda_m^{ik}} B_m^{ik} B_m^{ki*} N_{\nu}^{ij} N_{\nu}^{ji*} + \frac{\lambda_m^{ij}}{\lambda_m^{ik}} B_m^{ik} B_m^{ki*} M_{\nu}^{ij} M_{\nu}^{ji} \right] \left. \right\}
\end{aligned} \tag{A.9}$$

where

$$\begin{aligned}
E_{\mu n}^{ij} &= [M_{\mu}, A_n]^{ij} - (N_{\mu} B_n^*)^{ij} - (B_n N_{\mu}^*)^{ij} \\
E_{m\nu}^{ij} &= [A_m, M_{\nu}]^{ij} + (B_m N_{\nu}^*)^{ij} + (N_{\nu} B_m^*)^{ij} \\
F_{\mu n}^{ij} &= [(M_{\mu} B_n)^{ij} - (M_{\mu} B_n)^{ji}] - [(A_n N_{\mu})^{ij} - (A_n N_{\mu})^{ji}] \\
F_{m\nu}^{ij} &= [(A_m N_{\nu})^{ij} - (A_m N_{\nu})^{ji}] - [(N_{\nu} B_m)^{ij} - (N_{\nu} B_m)^{ji}].
\end{aligned} \tag{A.10}$$

From the action S'^{int} we have the diagram (a-1), (a-2) and (b-1) because the ghost terms are absent. The results are

$$\text{(a-1)} = -D_{ad} D_{as} J_1 + 3D_{ad} D_{as} J_2^- + 2D_{as} \tilde{J}_3 + 2D_{ad} J_3' \tag{A.11}$$

$$\text{(a-2)} = 2D_{ad} D_{as} K_1 + 4D_{as} (K_3^{(1)} + K_3'^{(1)}) + 4D_{ad} (K_3^{(2)} + \tilde{K}_3^{(2)}) \tag{A.12}$$

$$\text{(b-1)} = 4D_{as} L_1'^{(1)} + 4D_{as} L_1'^{(2)} + 2D_{as} L_2'^{(1)} + 2D_{ad} L_2^{(2)} \tag{A.13}$$

where

$$\begin{aligned}
J_1 &= \sum_{i,j,k \neq j} \frac{1}{2} \frac{1}{(\tilde{\lambda}^{ij})^2 (\tilde{\lambda}^{ik})^2}, \\
\tilde{J}_3 &= \sum_{i,j,k \neq j} \sum_{\mu} \frac{1}{2} \frac{\tilde{\lambda}_{\mu}^{ij}}{\tilde{\lambda}_{\mu}^{ik}} \frac{1}{(\tilde{\lambda}^{ij})^2 (\tilde{\lambda}^{ik})^2} (1 + \delta^{ik} - \delta^{ij}), \\
K_1 &= \sum_{i,j,k \neq j} \frac{1}{2} \frac{1}{(\lambda^{jk})^2 (\tilde{\lambda}^{ij})^2} (1 + \delta^{ik}), \\
K_3'^{(1)} &= \sum_{i,j,k \neq i} \sum_{\mu} \frac{1}{2} \frac{\lambda_{\mu}^{ik}}{\tilde{\lambda}_{\mu}^{ij}} \frac{1}{(\lambda^{ik})^2 (\tilde{\lambda}^{ij})^2} (1 - \delta^{ij}), \\
L_1'^{(1)} &= \sum_{i,j,k \neq i} \sum_{\mu} \frac{1}{2} \frac{(\lambda_{\mu}^{ik})^2}{(\lambda^{ik})^2} \frac{1}{(\tilde{\lambda}^{ij})^2 (\tilde{\lambda}^{kj})^2}, \\
L_1'^{(2)} &= \sum_{i,j,k \neq i} \sum_m \frac{1}{2} \frac{(\lambda_m^{ik})^2}{(\lambda^{ik})^2} \frac{1}{(\tilde{\lambda}^{ij})^2 (\tilde{\lambda}^{kj})^2}, \\
L_2'^{(1)} &= \sum_{i,j,k \neq i} \sum_{\mu} \frac{1}{2} \frac{\tilde{\lambda}_{\mu}^{ij} \tilde{\lambda}_{\mu}^{kj}}{(\lambda^{ik})^2 (\tilde{\lambda}^{ij})^2 (\tilde{\lambda}^{jk})^2} (1 - \delta^{ij} - \delta^{jk}). \tag{A.14}
\end{aligned}$$

Therefore

$$\begin{aligned}
W_2^{int}(\lambda) &= D_{ad} D_{as} J_1 - 3 D_{ad} D_{as} J_2^- - 2 D_{as} \tilde{J}_3 - 2 D_{ad} J_3' \\
&\quad - 2 D_{ad} D_{as} K_1 - 4 D_{as} (K_3^{(1)} + K_3'^{(1)}) - 4 D_{ad} (K_3^{(2)} + \tilde{K}_3^{(2)}) \\
&\quad - 4 D_{as} L_1'^{(1)} - 4 D_{as} L_1'^{(2)} - 2 D_{as} L_2'^{(1)} - 2 D_{ad} L_2'^{(2)}. \tag{A.15}
\end{aligned}$$

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